Tutorial 1: Ideal Gas, Electron Gas, Photon Gas

1. A box contains 1 mole of helium gas at room temperature and pressure.

(i) Find the average kinetic energy ε of the helium atoms.

(ii) Write down the formula for the total number of energy states below ε. Find this number.

(iii) Assuming that the atoms occupy these states only, find the probability that a state is occupied. Why is this only an estimate?

(iv) How does this result help us to find the most probable macrostate? State the macrostate.

(v) What are the main assumptions used?

2. One mole of silver is at 1 K. Each atom supplies one conduction electron.

(i) Assuming that the electrons behave like an ideal gas, write down the expression for the average kinetic energy of the electrons. Hence find the heat capacity.

(ii) At 1 K, the measured heat capacity is 0.5 mJ/K. Explain why it is different.

(iii) With the help of a graph, estimate the energy range of the electrons that are excited above the Fermi energy at temperature T.

(iv) Using the density of states $g(\varepsilon)$, derive an expression for the number n of excited electrons. (Molar volume of silver is 10.27 cm^3).

(v) Why is it reasonable to suppose that these electrons behave like the ideal gas?

(vi) Assuming they do, derive an expression for the heat capacity.

(vii) Find the Fermi energy. Calculate the heat capacity for silver at 1 K. Compare with the measured value and comment.

3.

(i) If a black body absorbs all radiation, what does it emit? Discuss using the idea of a photon gas.

(ii) Starting with the density of states in wavevector for an ideal gas, derive it in the frequency variable for photons.

(iii) Write down the probability that an energy state is occupied in the equilibrium photon gas. Comment on the difference from electrons.

(iv) Derive and sketch the frequency distribution of the number of photons.

(v) The wavelength at which this is maximum is given by Wien's displacement law, $\lambda_{max} = b/T$, where $b = 2.9\text{x}10^{-3}$ m.K. Estimate the temperature of the sun.

Speed of light in vacuum	c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
		=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12} Fm^{-1}
		=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge	e	=	1.60×10^{-19} C
Planck constant	h	=	6.63×10^{-34} Js
	$h/2\pi = \hbar$	=	1.05×10^{-34} Js
Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	=	1.38×10^{-23} JK^{-1}
Gas constant	R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant	m_u	=	1.66×10^{-27} kg
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	9.11×10^{-31} kg
Proton mass	m_p	=	1.67×10^{-27} kg
Gravitational constant	G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity	g	=	9.81 ms^{-2}
Bohr magneton	μ_B	=	9.27×10^{-24} JT^{-1}